CHAPTER 1

INTERNATIONAL METRIC SYSTEM (SI)

1. SCIENTIFIC NOTATION

The range of numbers that appear in the physical world is truly enormous:
mass of Earth ~ 5,980,000,000,000,000,000,000,000 kg
diameter of a proton ~ 0.000000000000001 m

So many zeros are inconvenient, and we employ a shorthand method of writing very large and very small numbers…
By using powers of 10, Earth's mass is more easily written as
5.98×10^{24} kg, and the diameter of a proton as 10^{-15} m.

2. THE INTERNATIONAL SYSTEM OF MEASUREMENTS

Measurement has always been one of man's most important activities. The sophistication of the measuring system is closely linked to the technological level of a society. Any measuring system can be expressed and built up in terms of four independent physical quantities:
1) Length
2) Mass
3) Time
4) Temperature

Three additional physical quantities are also made use of:
5) Electric current
6) Light intensity
7) Amount of substance

There is a need for standardization. You may wear a size 7 shoe in South Africa, but this unit would not be of much use if you were to

travel to Europe, where a different system is used; there, your shoe size would be 38.

Hundreds of years ago, people used what was readily available as standards for measurement, e.g. length measurements such as the foot. Over time, measurement systems have become more precise and more universal.

In the late 1700s, the French developed a logical and orderly system called the *metric system*. They defined the metre, the second, and the kilogram. The metric system consists of:

1) a set of standard units of measurement for distance, weight, volume, and so on, and
2) a set of prefixes that are used to express larger or smaller multiples of these units.

These prefixes represent multiples of 10 → decimal system

In 1889, an international organization called the General Conference on Weights and Measures was formed to meet periodically to refine these units of measure.

In 1960, this organization decided to modify the metric system by adopting a system of units called the

Système International d'Unités **→ SI**

SI Base Units

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Light intensity	candela	cd
Amount of substance	mole	mol

Other units, called derived units, are composed of combinations of basic units (e.g. the unit for speed, metres per second, is derived from the basic units metre and second).

The basic units were all carefully defined in such a way that, except for the kilogram, they are reproducible in any well-equipped laboratory. Occasionally one of the definitions is modified to make it more precise and more useful.

Length

The metre (m) was first defined as one ten-millionth (10^{-7}) of the distance between the North Pole and the equator along a meridian of the earth. In 1889, it was redefined as the distance between two finely engraved marks on a bar of platinum-iridium that was kept at exactly the freezing point of water (0ºC) in a vault outside Paris. In 1960, the standard of length was changed to depend upon an atomic constant – the wavelength of a particular orange-red light emitted by an isotope of krypton (^{86}Kr) gas. Because our ability (and need) to measure length has led us to require even greater accuracy, this standard also became insufficiently precise. Therefore, in 1983, the metre was redefined as the distance light travels in vacuum during 1/299,792,458 second.

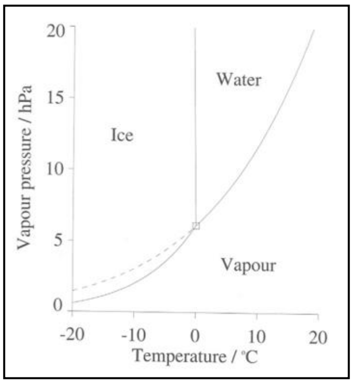
Time

The second (s) was originally defined as 1/86,400 of the mean solar day, which is the time interval, averaged over a year, from noon of one day to noon of the next. This definition is insufficient because Earth’s rotation is both slightly irregular and gradually slowing down from year to year. Therefore, in 1967, a definition of the second was adopted that depends on an atomic standard. The second is now defined as the duration of 9,192,631,770 periods of a particular vibration of caesium atom isotope (^{133}Cs). Clocks based on this standard are, in effect, identical because all atoms of ^{133}Cs are indistinguishable and because frequency can be measured in the laboratory to an accuracy of about four parts in 10^{13}.

Mass

The kilogram (kg) was originally defined as the mass of 1 litre of water under certain conditions of temperature and pressure. In 1901, the standard kilogram was defined as the mass of a particular cylinder of platinum-iridium alloy kept at the International Bureau of Weights and Measures in France. Duplicate copies of the cylinder made of this particularly stable alloy were kept in laboratories elsewhere. It has become apparent that the International Prototype Kilogram (IPK) or “*le Grand K*” was deteriorating so that the accuracy of the kilogram is within an order of 10^{-8} parts. In 2019 it was redefined based on an invariant, fundamental physical constant, in particular Planck’s constant (h). This fixes the value of the kilogram in terms of the second and the metre, and eliminates the need for the IPK. The new definition only became possible with the advent of new instruments, such as the Kibble balance, which can measure Planck’s constant with sufficient accuracy. Planck’s constant is now redefined as exactly $6.62607015 \times 10^{-34}$ kg.m^2.s^{-1}. Since the metre is defined in terms of the speed of light in a vacuum, the kilogram is now defined in terms of time only.

Temperature



The Kelvin (K) is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. Thus, at the triple point of water the thermodynamic temperature is 273.16 K. This definition is unsatisfactory for temperatures below 20 K and above 1300 K. The Kelvin scale is an absolute, thermodynamic temperature scale using as its null point absolute zero, the temperature at which all thermal motion ceases. The temperature 0 K is commonly referred to as "absolute zero". Note that we don't say "degree Kelvin" or use °K. In 2019 the Kelvin was redefined by fixing the numerical value of the Boltzmann constant (*k*) at 1.380649×10^{-23} $J.K^{-1}$ (or $kg.m^{2}.s^{-2}.K^{-1}$), where the new definition of the kilogram applies.

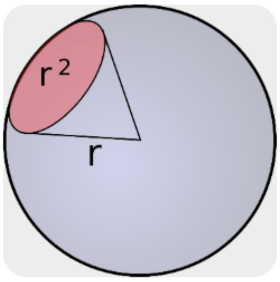
Amount of Substance

The mole (mol) was originally defined as the amount of substance of a system which contains as many elementary entities (e.g. molecules or atoms) as there are atoms in 12 grams of carbon-12 (^{12}C). In 2019 the mole was redefined. One mole contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance (*n*) of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles. The previous definition of the mole linked it to the kilogram. The revised definition breaks that link by making a mole a specific number of entities of the substance in question.

Light Intensity

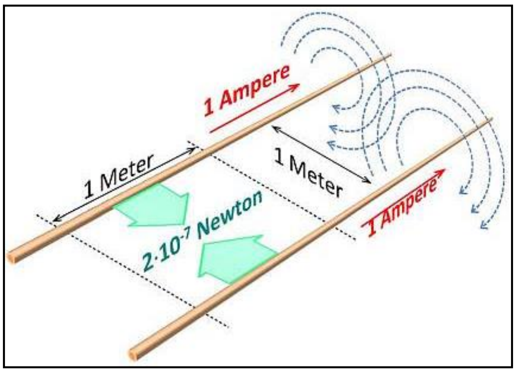
Light intensity was originally related to the light emitted by a burning candle of specified properties, or candlepower (cp). The specified properties, however, was redefined from time to time. In 1948, the candela (cd) replaced candlepower, with 1 cp = 0.981 cd. In general modern use, a candlepower equates directly (1:1) to the candela. The candela is defined as the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 × 10^{12} Hz (hertz) and that has a radiant intensity in that direction of 1/683 W.sr^{-1} (watt per steradian).

Note: the steradian or square radian is the SI unit of solid angle. It is used in three-dimensional geometry, and is analogous to the radian, which quantifies planar angles. Whereas an angle in radians, projected onto a circle, gives a length on the circumference, a solid angle in steradians, projected onto a sphere, gives an area on the surface.



Electric Current

The ampere was originally defined as that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in a vacuum, would produce between these conductors a force equal to 2 × 10^{-7} N per metre of length. In 2019 the ampere was redefined by taking the fixed numerical value of the elementary charge (*e*) to be 1.602176634 × 10^{-19} C (or A.s). The old definition was difficult to realise with high precision in practice, while the new one is more intuitive and easier to realise and now only depends on the definition of the second.



The SI system of units, also known as the *metric system* or *mks system* (after *m*etre, *k*ilogram, and *s*econd) is by far the most important and widely accepted system used in the world today.

SI-prefixes

The use of SI base units only may sometimes lead to uncomfortably large or small numbers. For example:

1) The distance between Johannesburg and Durban would be 657 000 m or 6.57×10^5 m
2) The wavelength of green light is 0.000 000 51 m or 5.1×10^{-7} m

Neither of these quantities is convenient for everyday use. In order to avoid this cumbersome method of writing, a useful set of prefixes in SI can be used to replace certain powers of ten. These prefixes are listed in the table below. Unfortunately, there are two different sets of naming conventions used around the world, the so-called short and long scales. The short scale is based on powers of one thousand and is used in most English- and Arabic-speaking countries, Brazil and several other countries. The long scale is based on powers of one million and is used throughout continental Europe and most French- and Spanish-speaking countries. South Africa uses both the long scale (in Afrikaans and sometimes English) and the short scale (in English).

Unit Prefixes for Powers of 10

Prefix	Symbol	Multiple	Amount	English word	
				Short	Long
yotta	Y	10^{24}	1 000 000 000 000 000 000 000 000	septillion	quadrillion
zetta	Z	10^{21}	1 000 000 000 000 000 000 000	sextillion	trilliard
exa	E	10^{18}	1 000 000 000 000 000 000	quintillion	trillion
peta	P	10^{15}	1 000 000 000 000 000	quadrillion	billiard
tera	T	10^{12}	1 000 000 000 000	trillion	billion
giga	G	10^{9}	1 000 000 000	billion	milliard
mega	M	10^{6}	1 000 000	million	million
kilo	k	10^{3}	1 000	thousand	thousand
hecto	h	10^{2}	100	hundred	hundred
deka	da	10^{1}	10	ten	ten
deci	d	10^{-1}	0.1	tenth	tenth
centi	c	10^{-2}	0.01	hundredth	hundredth
milli	m	10^{-3}	0.001	thousandth	thousandth
micro	µ	10^{-6}	0.000 001	millionth	millionth
nano	n	10^{-9}	0.000 000 001	billionth	milliardth
pico	p	10^{-12}	0.000 000 000 001	trillionth	billionth
femto	f	10^{-15}	0.000 000 000 000 001	quadrillionth	billiardth
atto	a	10^{-18}	0.000 000 000 000 000 001	quintillionth	trillionth
zepto	z	10^{-21}	0.000 000 000 000 000 000 001	sextillionth	trilliardth
yokto	y	10^{-24}	0.000 000 000 000 000 000 000 001	septillionth	quadrillionth

1) The prefix *kilo*-, which stands for 10^3, can now be used to state the distance between Johannesburg and Durban as
   657 kilometres (km)
2) The prefix *nano*-, which stands for 10^{-9}, can now be used to state the wavelength of green light as 510 nanometres (nm)

3. ACCURACY AND SIGNIFICANT FIGURES

Uncertainty in measurement

Physics rests on experiment, and experiment requires measurement. Every measurement involves error. An *uncertainty* is an indication of the accuracy of a measurement. The uncertainty depends on the

accuracy and calibration of the instrument that is making the measurement and on how well the instrument can be read.

Example:

Suppose we want to measure the length of a book, using a ruler divided into millimetres. You measure 294 mm or 29.4 cm, your fellow students measure 29.3 cm and 29.5 cm respectively. Notice that everyone agrees on the first two digits but the third is known only approximately. It is a doubtful figure. Here it would be correct to say that the length of the book is 29.4 ± 0.1 cm.

Here, 29.4 cm is called the *central value* and 0.1 cm the *uncertainty* around that central value.

In this case, the basis of the uncertainty lies in how well our eyes can read the ruler and on the precision with which the ruler was made.

The term *percentage uncertainty* is often used as a measure of the ratio of the uncertainty of a quantity to its central value. The percentage uncertainty is found by multiplying the ratio by 100. The percentage uncertainty of our length measurement is thus

$$\left(\frac{0.1cm}{29.4cm}\right)100 = 0.3\%$$

If we measure the width of the book to be 20.9 ± 0.1 cm, the surface area can be calculated to be 29.4 cm × 20.9 cm = 614 cm^2 by multiplying the length by the width. Because the measurements of length and width both contain uncertainties, the area is also uncertain. When two quantities are multiplied or divided, the net uncertainty is given by $\sqrt{P_1^2 + P_2^2}$ where P_1 and P_2 are the respective *percentage uncertainties* of the two quantities. In the case of the book the uncertainty would then be given by $\sqrt{0.3^2 + 0.5^2} = 0.6\%$. This means an uncertainty of (614 cm^2)(0.006) = 4 cm^2, and the area of the book is 614 ± 4 cm^2.

Significant figures

Recall our length measurement of 29.4 ± 0.1 cm. Notice that the first two digits are exactly known but the third digit is a doubtful one. The digits that are exactly known and the first doubtful digit are called *significant figures*.

We imply a certain degree of uncertainty in a quantity when we assign a certain number of digits to its numerical value.

Thus, when we say an object is 1.00 m long, we mean that it is between 0.995 m and 1.0005 m long. If we wanted to say that the length is somewhere between 0.9995 m and 1.0005 m, we would say that the length is 1.000 m. In the first case, 3 significant figures are used to describe the object's length; in the second case, the number of significant figures is 4.

Zeros that are used only to set a decimal point are not part of our count of significant figures. Thus 0.00075 has 2 significant figures, not 6.

To take a more extreme example, we mentioned that the mass of Earth is 5,980,000,000,000,000,000,000,000 kg. Surely we do not know Earth's mass to 25 significant figures!

Scientific notation provides a way to avoid this ambiguity.

When we write the mass of Earth as 5.98×10^{24} kg, we indicate unambiguously that we know the mass to 3 significant figures; if we knew the mass to only 2 significant figures, we would write

6.0×10^{24} kg.

WHEN WE USE DATA IN CALCULATIONS WE MUST NOT INTRODUCE MORE DIGITS INTO THE RESULT THAN THE ORIGINAL DATA ALLOWS.

Carry a calculation only to as many significant figures as are contained in the input parameter with the fewest significant figures.

For example, 11.0 has 3 significant figures and 3.0 has 2 significant figures, so the ratio of 3.0 to 11.0 can only have 2 significant figures.

$\therefore \left(\frac{3.0}{11.0}\right) = 0.27$ or 2.7×10^{-1} and <u>not</u> 0.27272727….

Your handheld calculator will tempt you to write the latter!

For the purpose of this course, always use scientific notation and round answers off to no more than TWO decimal places

Examples:

54 m must be written as 5.4×10^{1} m

546 m must be written as 5.46×10^{2} m

546892 m must be written as 5.47×10^{5} m

0.000546892 m must be written as 5.47×10^{-4} m

0.54 m must be written as 5.4×10^{-1} m

4. DIMENSIONAL ANALYSIS

There are three basic ways to describe any physical quantity:

- the space it takes up,
- the matter it contains, and
- how long it persists.

All measurements can be reduced ultimately to the measurement of length, time, and mass. Any physical quantity, no matter how complex, can usually be expressed as an algebraic combination of these three basic quantities. Length, time and mass therefore specify the three *primary dimensions*.

We use the abbreviations [L], [T] and [M] for these primary dimensions.

Any physical quantity has dimensions that are algebraic combinations $[L^{q}T^{r}M^{s}]$ of the primary dimensions, where the superscripts *q*, *r*, and *s*

refer to the order (or power) of the dimension. For example, the dimension of speed v is [L/T] or [LT^{-1}] while an area has dimension [L^2]. If all the exponents q, r, and s are zero, the combination will be dimensionless. The exponents q, r, and s can be positive integers, negative integers, or even fractional powers.

Do not confuse the dimension of a quantity with the units in which it is measured.

A speed may have units of meters per second, miles per hour, or, for that matter, light-years per century. All of these different choices of units are consistent with the dimension [LT^{-1}].

5. UNIT CONSISTENCY AND CONVERSIONS

Units are multiplied and divided just like ordinary algebraic symbols. This fact gives us an easy way to convert a quantity from one set of units to another. The key idea is that we can express the same quantity in two different units and form equality.

To make converting from one unit to another simpler, we will use a method called the *factor-unit method*. With this approach, any problem that requires conversion from one unit to another can be set up and solved in a similar manner. We can say that:

Quantity wanted = Quantity given × Factor unit

When you multiply the quantity given by the proper factor unit, some of the units cancel to give the desired quantity.

Example: convert metres to centimetres

Quantity wanted = Quantity given × Factor unit

$$centimetres = metre \times \frac{centimetres}{metre}$$

Note that the factor unit expresses a relationship between the quantity wanted and the quantity given.

The factor unit is written in such a way that the given units cancel when you multiply the quantity given times the factor unit.

Of course any factor unit can be expressed in two ways.

For example, the relationship 1 metre = 100 centimetres can be expressed as:

$$\frac{100 centimetres}{1 metre} \quad \text{or} \quad \frac{1 metre}{100 centimetres}$$

You must choose the factor unit that will make the proper terms cancel.

Problem 1: Convert 40 m to cm

Solution: There are 100 centimetres per metre → $\frac{100 centimetres}{1 metre}$

$$? centimetres = 40 metres \times \frac{100 centimetres}{1 metre} = 4000 cm$$

Thus, there are 4000 centimetres in 40 metres

Problem 2: Convert 430 milligrams to grams

Solution: There are 1000 milligrams per gram → $\frac{1 gram}{1000 milligrams}$

$$? grams = 430 mg \times \frac{1 g}{1000 mg} = 0.43 g$$

Problem 3: Convert 580 millimetres to centimetres

Solution: There are 100 cm per m → $\frac{100 cm}{1 m}$

There are 1000 mm per m → $\frac{1000 mm}{1 m}$ or $\frac{1 m}{1000 mm}$

$$\frac{100 cm}{1 m} \times \frac{1 m}{1000 mm} = \frac{100 cm}{1000 mm} = \frac{1 cm}{10 mm}$$

Thus, there are 10mm per cm

$$?cm = 580mm \times \frac{1cm}{10mm} = 58cm$$

There are 58 cm in 580 mm

NB: a little reasoning or common logic will go a far way…

Problem 4: Convert 2.3 kg to the corresponding mass in

a) grams b) decigrams

c) centigrams d) milligrams

Solution:

a) There are 1000 g per kg → $\frac{1000g}{kg}$

$$?g = 2.3kg \times \frac{1000g}{kg} = 2300g$$ (2.30×10^3 g)

b) There are 10 dg per g → $\frac{10dg}{g}$

$$?dg = 2300g \times \frac{10dg}{g} = 23000dg$$ (2.30×10^4 dg)

c) There are 100 cg per g → $\frac{100cg}{g}$

$$?cg = 2300g \times \frac{100cg}{g} = 230000cg$$ (2.30×10^5 cg)

d) There are 1000 mg per g → $\frac{1000mg}{g}$

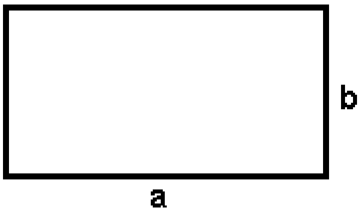
$$?mg = 2300g \times \frac{1000mg}{g} = 2300000mg$$ (2.30×10^6 mg)

6. GEOMETRIC FORMULAS

- Rectangle with length *a* and width *b*

  Circumference $= 2a + 2b$

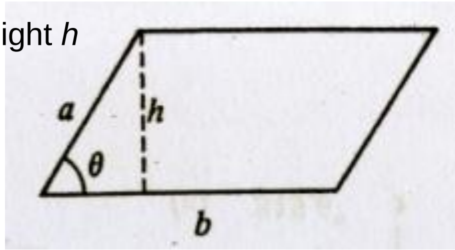
  Area $= ab$



- Parallelogram with base *b* and height *h*

  Circumference $= 2a + 2b$

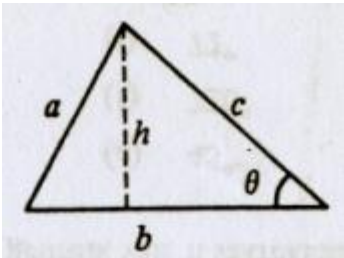
  Area $= bh = ab\sin\theta$



- Triangle with base *b* and height *h*

  Circumference $= a + b + c$

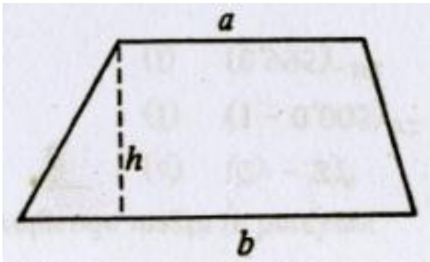
  Area $= \frac{1}{2}bh = \frac{1}{2}bc\sin\theta$



- Trapezium with parallel sides *a* and *b* and height *h*

  Circumference $= a + b + c + d$

  Area $= \frac{1}{2}(a + b)h$

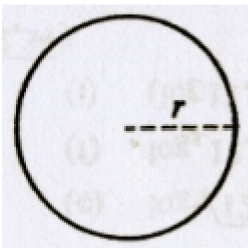


- Circle with radius *r*

  Diameter $D = 2r$

  Circumference $= \pi D = 2\pi r$

  Area $= \frac{1}{4}\pi D^2 = \pi r^2$

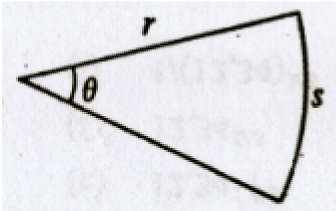


- Circle segment with radius *r*

  Arc length $s = r\theta$

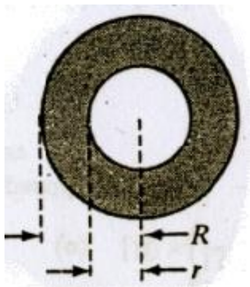
  Circumference $= 2r + r\theta$

  Area $= \frac{1}{2} rs = \frac{1}{2} r^2 \theta$



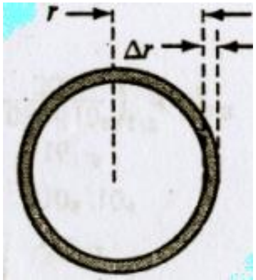
- Ring/Annulus with inner radius *r* and outer radius *R*

  Area $= \pi(R^2 - r^2) = \pi(R + r)(R - r)$



- Thin ring/annulus with radius *r* and width *Δr*

  Area $\approx circumference \times width$
  $\approx 2\pi r \Delta r$



- Rectangular parallelepiped with sides *a*, *b* and *c*

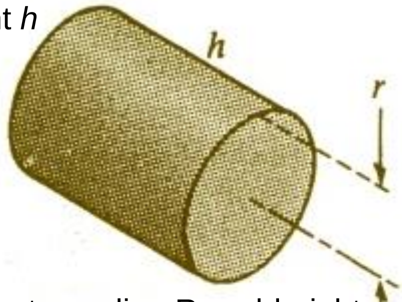
  Area $= 2(ab + bc + ca)$

  Volume $= abc$

- Solid circle cylinder with radius *r* and height *h*

  Area $= 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$

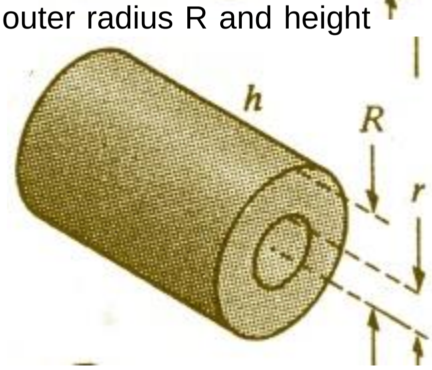
  Volume $= \pi r^2 h$



- Hollow circle cylinder with inner radius *r*, outer radius R and height *h*

  Area $= 2\pi(R^2 - r^2) + 2\pi h(R + r)$

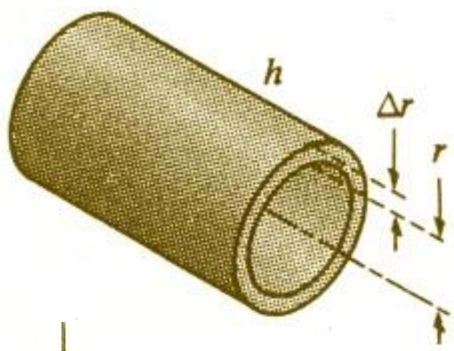
  Volume $= \pi(R^2 - r^2)h$



- Hollow circle cylinder with radius *r*, height *h* and a thin wall of width *Δr*

  Area $\approx 4\pi rh$

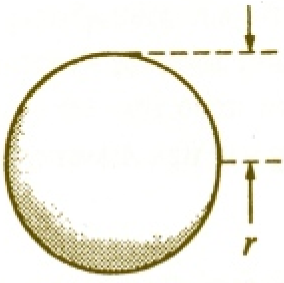
  Volume $\approx 2\pi rh\Delta r$



- Sphere with radius *r*

  Area $= 4\pi r^2$

  Volume $= \frac{4}{3}\pi r^3$



- Circular cone with radius *r* and height *h*

  Area $= \pi rl + \pi r^2$

  Volume $= \frac{1}{3}\pi r^2 h$